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Hadronic modes in the quark plasma with an internal symmetry

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Abstract. We show that requiring the quark partition function to be color singlet in the SU(3) color gauge group leads to reordering of the thermodynamic potential in terms of the colorless multiquark modes ($q\bar{q}$, qqq, $\bar{q}\bar{q}\bar{q}$, $\cdots$) at any given temperature. These color-singlet structures are not bound states in a real sense, rather they are a combination of constituent quarks only. In accord with the "preconfinement" property of QCD, under a suitable confining mechanism, these could evolve into color-singlet hadrons/baryons at low temperatures. At fairly high temperatures, these multiquark color-singlet structures exist in the plasma as hadronic modes, just as in the more familiar low-temperature phase. This suggests that there exists a strong color correlation in the plasma at all temperatures.

The success of the quark model, quantum chromodynamics (QCD), and the nonobservability of the free partons (q, $\bar{q}$, g) has entailed the concept of confinement. QCD, the theory of strong interactions, is not perturbative at large distances. Thus, the confinement itself can not be treated perturbatively. There are reasons to believe that the partons confined inside hadrons may not survive collisions between heavy nuclei at relativistic energies [1]. One of the most interesting predictions of QCD at high temperature is the transition from the confined/chirally broken phase to the deconfined/chirally symmetric state of quasi-free quarks and gluons, the so-called quark–gluon plasma. At very high temperatures, the bulk properties (e.g., energy density, pressure, and entropy) of QCD matter seem to be described by a gas of nearly free quarks and gluons. However, it is also known that long-range, nonperturbative effects disrupt this simple picture even at fairly high temperatures [2].

Lattice calculations [3] have provided ample evidence that the long-distance behavior of the high-temperature phase is characterized by the propagation of color-singlet objects like multiquark structures. The determination of the plasma screening length shows evidence of a kind of correlation in the quark–gluon plasma at all temperatures. Now what these structures are and how they show up is not yet theoretically clear. Quite some time ago, an indication was made as a "precursor" to the confinement property of QCD by Amati and Veneziano [4] where the cascading and fragmenting partons produced in hadronic collisions rearrange themselves into color-singlet clusters that ultimately evolve into hadrons [5,6]. These considerations convince us that it is important to incorporate the *dynamic* requirement of color singletness of the quark matter to take into account such interactions that "tunnel" into hadronic matter phase space [6].

The purpose of the present study is to reconsider the statistical themodynamical description of quantum gases consisting of quarks and antiquarks in such a way that the underlying symmetry can amount to reordering of thermodynamic potential in terms of the color-singlet multiquark modes at any temperatures. We note that the ingredients of our rather simple calculation have been around for more than a decade[7–11], but to our knowledge, no one has checked these dramatic consequences explicitly before.

We begin with the partition function for a quantum gas, containing quarks and antiquarks within a finite volume, which can be written as

$$\mathcal{Z} = \mathrm{Tr}\left(\hat{\mathcal{P}}\exp(-\beta\hat{H})\right) , \tag{1}$$

where $\beta = 1/T$ is the inverse of temperature, $\hat{H}$ is the Hamiltonian of the physical system, and $\hat{\mathcal{P}}$ is the projection operator with respect to any configuration admitted by a system. Now, for a symmetry group $\mathcal{G}$ (compact Lie group) having unitary representation $\hat{U}(g)$ in a Hilbert space $\mathcal{H}$, the projection operator can be written as [12]

$$\hat{\mathcal{P}}_j = d_j \int_{\mathcal{G}} \mathrm{d}\mu(g)\chi_j^\star(g)\hat{U}(g) , \tag{2}$$

where d_j and χ_j are the dimension and the character, respectively, of the irreducible representation j of $\mathcal{G}$. $\mathrm{d}\mu(g)$ is the normalized Haar measure in the group $\mathcal{G}$. The symmetry group associated with the color-singlet configuration

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of the system is $SU(N_C)$, N_C is the number of color corresponding to fundamental representation. For the $SU(N_C)$ color-singlet configuration, $d_j = 1$ and $\chi_j = 1$. The explicit form of the Haar measure corresponding to $SU(N_C)$ can be found in [10] as

$$\int_{SU(N_C)} d\mu(g) = \frac{1}{N_C!}\left(\prod_{c=1}^{N_C-1}\int_{-\pi}^{\pi}\frac{d\theta_c}{2\pi}\right) \left[\prod_{i<k}^{N_C}\left(2\sin\frac{\theta_i-\theta_k}{2}\right)^2\right] , \quad (3)$$

where θ_l is a class parameter obeying the periodicity condition $\sum_{l=1}^{N_C}\theta_l = 0\ (\mathrm{mod} 2\pi)$, which ensures that the group element is $SU(N_C)$. Now the partition function for the system becomes

$$\mathcal{Z} = \int_{SU(N_C)} d\mu(g)\mathrm{Tr}\left(\hat{U}(g)\exp(-\beta\hat{H})\right) . \quad (4)$$

The Hilbert space $\mathcal{H}$ of the composite system has the structure of a tensor product of the individual Fock spaces:

$$\mathcal{H} = \mathcal{H}_q \otimes \mathcal{H}_{\bar{q}} , \quad (5)$$

where the subscripts q and $\bar{q}$ denote the quark and antiquark. Now the trace involved in (4) decomposes into the product of two traces as

$$\mathcal{Z} = \int_{SU(N_C)} d\mu(g)\ \mathrm{Tr}\left(\hat{U}_q(g)\exp(-\beta\hat{H}_q)\right) \mathrm{Tr}\left(\hat{U}_{\bar{q}}(g)\exp(-\beta\hat{H}_{\bar{q}})\right) . \quad (6)$$

The traces involved in (6) can now be evaluated by the use of the known results for fermions (for details, see [10]), yielding

$$\mathcal{Z} = \int_{SU(N_C)} d\mu(g)\exp(\Theta) , \quad (7)$$

with

$$\Theta = \mathrm{tr}\Big[\ln\left(1+e^{i\theta_c}e^{-\beta(\epsilon_\alpha^q-\mu^q)}\right) + \ln\left(1+e^{-i\theta_c}e^{-\beta(\epsilon_\alpha^{\bar{q}}-\mu^{\bar{q}})}\right)\Big] , \quad (8)$$

where the trace indicates the summation over color (c), flavor (q), spin (s) and single-particle states (α) with $\epsilon_\alpha = \sqrt{p_\alpha^2+m^2}$. μ^q and $\mu^{\bar{q}}$ are, respectively, quark and antiquark chemical potential. In (8) one can replace $\epsilon^{\bar{q}}$ by ϵ^q, and $\mu^{\bar{q}}$ by $-\mu^q$. It is worth noting here that (7) represents the general structure of the color-projected partition function that exists in the literature [7–11], and depending upon the nature of the problem, it has been utilized accordingly. For convenience, we make a substitution $\xi^q = -i\beta\mu^q$ and after a little algebra, (8) becomes

$$\Theta = \ln\Big\{\prod_\alpha\prod_c^{N_C}\prod_q^{N_f}\prod_s^{N_s}\Big[e^{-\beta\epsilon_\alpha^q} \times (2\cosh\beta\epsilon_\alpha^q + 2\cos(\theta_c+\xi^q))\Big]\Big\}. \quad (9)$$

where N_f and N_s are the numbers for flavor and spin degrees of freedom, respectively. Upon substitution of (9) into (7), we get, for a finite volume,

$$\mathcal{Z} = \prod_\alpha\int_{SU(N_C)} d\mu(g)\Big\{\prod_c^{N_C}\prod_q^{N_f}\prod_s^{N_s}\Big[e^{-\beta\epsilon_\alpha^q} \times (2\cosh\beta\epsilon_\alpha^q + 2\cos(\theta_c+\xi^q))\Big]\Big\}. \quad (10)$$

We would like to point out that the product over α is written outside the group integration without any loss of generality. For exact flavor symmetry, (10) can be written as

$$\mathcal{Z} = \prod_\alpha\int_{SU(N_C)} d\mu(g)\Big\{\prod_c^{N_C}\Big[e^{-\beta\epsilon_\alpha} \times (2\cosh\beta\epsilon_\alpha + 2\cos(\theta_c+\xi))\Big]^{2N_f}\Big\}. \quad (11)$$

For $N_C = 3$ and $N_f = 2$, the above equation becomes

$$\mathcal{Z} = \prod_\alpha\int_{SU(3)} d\mu(g)\Big\{2^{12}e^{-12\beta\epsilon_\alpha} \times[\cosh\beta\epsilon_\alpha + \cos(\theta_1+\xi)]^4 \times[\cosh\beta\epsilon_\alpha + \cos(\theta_2+\xi)]^4 \times[\cosh\beta\epsilon_\alpha + \cos(\theta_1+\theta_2-\xi)]^4\Big\} , \quad (12)$$

with the measure corresponding to SU(3) color symmetry obtained from (3) as

$$\int_{SU(3)} d\mu(g) = \frac{8}{3\pi^2}\int_{-\pi}^{\pi}d\theta_1\int_{-\pi}^{\pi}d\theta_2\ \sin^2\frac{\theta_1-\theta_2}{2} \times\sin^2\frac{\theta_1+2\theta_2}{2}\ \sin^2\frac{2\theta_1+\theta_2}{2} , \quad (13)$$

where we have made use of the periodicity condition $\sum_{l=1}^{N_C=3}\theta_l = 0$. Substituting (13) into (12), and performing the several hundred elementary integrations in the group space, one can write the color-singlet thermodynamic potential for finite system as

$$\Omega = -\frac{1}{\beta}\sum_\alpha\ln\Big[1+M+B\Big] , \quad (14)$$

where M corresponds to mesonic modes and is given as

$$M = 16e^{-2\beta\epsilon_\alpha} + 136e^{-4\beta\epsilon_\alpha} + 816e^{-6\beta\epsilon_\alpha} + 1616e^{-8\beta\epsilon_\alpha} + 4941e^{-10\beta\epsilon_\alpha} + 6160e^{-12\beta\epsilon_\alpha} + 4941e^{-14\beta\epsilon_\alpha} + 1616e^{-16\beta\epsilon_\alpha} + 816e^{-18\beta\epsilon_\alpha} + 136e^{-20\beta\epsilon_\alpha} + 16e^{-22\beta\epsilon_\alpha} + e^{-24\beta\epsilon_\alpha} , \quad (15)$$

whereas those of baryonic (antibaryonic) modes are obtained as

$$B = 20e^{-3\beta(\epsilon_\alpha\mp\mu)} + 180e^{-2\beta\epsilon_\alpha}\ e^{-3\beta(\epsilon_\alpha\mp\mu)} + 816e^{-4\beta\epsilon_\alpha}\ e^{-3\beta(\epsilon_\alpha\mp\mu)} + 2320e^{-6\beta\epsilon_\alpha}\ e^{-3\beta(\epsilon_\alpha\mp\mu)}$$

$$
\begin{aligned}
&+ 3020\mathrm{e}^{-8\beta\epsilon_\alpha}\ \mathrm{e}^{-3\beta(\epsilon_\alpha \mp\mu)} + 3020\mathrm{e}^{-10\beta\epsilon_\alpha}\ \mathrm{e}^{-3\beta(\epsilon_\alpha \mp\mu)} \\
&+ 2320\mathrm{e}^{-12\beta\epsilon_\alpha}\ \mathrm{e}^{-3\beta(\epsilon_\alpha \mp\mu)} + 816\mathrm{e}^{-14\beta\epsilon_\alpha}\ \mathrm{e}^{-3\beta(\epsilon_\alpha \mp\mu)} \\
&+ 180\mathrm{e}^{-16\beta\epsilon_\alpha}\ \mathrm{e}^{-3\beta(\epsilon_\alpha \mp\mu)} + 20\mathrm{e}^{-18\beta\epsilon_\alpha}\ \mathrm{e}^{-3\beta(\epsilon_\alpha \mp\mu)} \\
&+ 50\mathrm{e}^{-6\beta(\epsilon_\alpha \mp\mu)} + 240\mathrm{e}^{-2\beta\epsilon_\alpha}\ \mathrm{e}^{-6\beta(\epsilon_\alpha \mp\mu)} \\
&+ 570\mathrm{e}^{-4\beta\epsilon_\alpha}\ \mathrm{e}^{-6\beta(\epsilon_\alpha \mp\mu)} + 800\mathrm{e}^{-6\beta\epsilon_\alpha}\ \mathrm{e}^{-6\beta(\epsilon_\alpha \mp\mu)} \\
&+ 570\mathrm{e}^{-8\beta\epsilon_\alpha}\ \mathrm{e}^{-6\beta(\epsilon_\alpha \mp\mu)} + 240\mathrm{e}^{-10\beta\epsilon_\alpha}\ \mathrm{e}^{-6\beta(\epsilon_\alpha \mp\mu)} \\
&+ 50\mathrm{e}^{-12\beta\epsilon_\alpha}\ \mathrm{e}^{-6\beta(\epsilon_\alpha \mp\mu)} \\
&+ 20\mathrm{e}^{-9\beta(\epsilon_\alpha \mp\mu)} + 40\mathrm{e}^{-2\beta\epsilon_\alpha}\ \mathrm{e}^{-9\beta(\epsilon_\alpha \mp\mu)} \\
&+ 40\mathrm{e}^{-4\beta\epsilon_\alpha}\ \mathrm{e}^{-9\beta(\epsilon_\alpha \mp\mu)} + 20\mathrm{e}^{-6\beta\epsilon_\alpha}\ \mathrm{e}^{-9\beta(\epsilon_\alpha \mp\mu)} \\
&+ \mathrm{e}^{-12\beta(\epsilon_\alpha \mp\mu)}\ . \qquad (16)
\end{aligned}
$$

In the infinite volume limit, the $\sum_\alpha$ in (14) can be replaced by integration as

$$\Omega = -\frac{1}{\beta} \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\ \ln\left[1 + M + B\right]\ . \qquad (17)$$

It should be noted here that in (15) and (16), one needs to replace ϵ_α by just ϵ.

Equation (17) clearly shows that the color projection amounts to reordering the thermodynamic potential in terms of Boltzmann factors for the colorless multiquark (mesonic/baryonic) modes at any temperature. The mesonic modes can be seen in (15) to have quark content $\mathrm{q}^n\bar{\mathrm{q}}^n,\ n = 1, \cdots, 12$ with energy $2n\epsilon$. On the other hand, baryonic (antibaryonic) modes are very transparent from (16), with quark content $\mathrm{q}^{m+N_\mathrm{C}B}\bar{\mathrm{q}}^m\ \ (\bar{\mathrm{q}}^{m+N_\mathrm{C}\bar{B}}\mathrm{q}^m)$; $B(\bar{B}) = 1, 2, 3, 4, \cdots$ is the baryon (antibaryon) number, $m = 0, 1, \cdots$ (restricted by the values of $B(\bar{B})$; see (16)), and $N_\mathrm{C} = 3$. In general, the energy of these baryonic (antibaryonic) modes can be written as $2m\epsilon + BN_\mathrm{C}(\epsilon \mp \mu)$. The maxim values of n, m, and $B(\bar{B})$ depend on the number of flavor chosen (see, e.g., (11)). The interesting feature of this color projection is that the chemical potential, μ, always appears with the color factor N_C in the baryonic Boltzmann factor. Of course, these color-singlet structures are not bound states in a real sense; rather, we should say that they are a combination of constituent quarks only.

Under a suitable confining mechanism, we hope that these multiquark colorless structures could evolve into color-singlet hadrons in the low-temperature limit. This is in accord with the "preconfinement" property of QCD noted by Amati and Veneziano [4] quite some time ago. In the mesonic sector, $n = 1$ corresponds to low-lying mesons (the first term in (15)) whereas $n > 1$ represents exotic mesons. It is to be noted that the factor of 16 appearing in the first term in (15) can amount to 16 low-lying degenerate mesonic states corresponding to SU(2) flavor and SU(2) spin symmetry. We would like to point out here that the color-singlet thermodynamic potential can possibly extract mesonic states in which pions, being the pseudoscalar Goldstone boson, could be the exception. On the other hand, $m = 0$, $B(\bar{B}) = 1$, and $N_\mathrm{C} = 3$ amount to low-lying baryons (the first term in (16)) whereas $m > 1$, $B(\bar{B}) \geq 1$, and $N_\mathrm{C} = 3$ correspond to exotic baryons. The factor of 20 appearing in the first term in (16) represents 20 baryonic and 20 antibaryonic states. This is also quite consistent with SU(2) flavor and SU(2) spin symmetry in which nucleons and deltas are degenerate, and their total degeneracy is 40. It is also apparent that, as is expected physically, the low-lying hadronic modes play a dominant role at low temperature, while the exotic hadronic modes/collective modes are most relevant at high temperature. In a nonsupersymmetric tachyonless string model, Kutasov and Seiberg [13] suggested that the number of fermionic and bosonic states must approach each other as increasingly massive states are included in the hadronic density of states. On the basis of this result, Freund and Rosner [14] have proposed that there must exist exotic mesons and baryons with similar quark content to that discussed above to have equalization of mesonic and baryonic density of states, since the observed states are deficient at a higher mass range (for the meson, it is 1.3 GeV and for baryon, 2 GeV). In the low-temperature limit, the existence of these exotic hadrons is highly improbable, but they may start to appear with an increase in temperature. In a model-dependent calculation [15], the limiting temperature has been found to vary with the mass of the hadrons.

As was discussed above, the multiquark color-singlet modes become very relevant at very high temperatures. The estimation of energy density and pressure [9,11] of the system at moderately high temperatures shows deviation from its ideal gas behavior. In this context, it has been suggested [3] that the long-distance behavior of the high-temperature phase can be characterized by the propagation of color-singlet multiquark structures. In this simple-minded calculation, such color-singlet objects appear very cleanly at high temperature. Now the question becomes: What do these modes correspond with at high temperature? As we see, these multiquark objects are not real bound states, but just a combination of constituent quarks indistinguishable from the color-singlet hadrons at low temperature. This could lead to a kind of analog of the high-temperature phase to that of low temperature, and we naively speculate that if there is a phase transition in light-quark QCD, it may be of a chiral character rather than a deconfinement character. However, this issue requires a more careful investigation.

Finally, we would like to comment here on one important aspect of the three-flavor case. If one considers three flavors (for which calculation will be cumbersome), there will be a structure consisting of 6 quarks (two of each flavor) which could be of particular interest to those in the community searching for strangelets in heavy-ion collisions. This particular structure is known as quark-alpha (Q_α) in the literature [16] and could be a probable candidate for detecting strangelets in heavy ion collisions.

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